A Proposal for the Classification of Mathematical Sculpture

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Abstract

This paper summarises my research on mathematical sculpture and offers the latest version of my proposed classification. The introduction to the study of this kind of sculpture in higher education requires a taxonomy to classify mathematically all its different types. From our point of view, this taxonomy had never been arranged in depth. The first version of the classification (with Professor Javier Barrallo) was shown at ISAMA BRIDGES 2003 *Meeting Alhambra* [1]. It was a preliminary starting point in which we suggested nine categories, based on mathematical concepts. Since then, I have improved taxonomy structure until it has adapted to a mathematical classification. The best way for categorising mathematical sculpture consists in establishing general groups for the different areas of Mathematics and then subdividing these groups according to the main mathematical concepts used in the sculpture design. As it has happened in the past, I would be pleased to receive suggestions from the art and mathematics community in order to improve this taxonomy.

1. Introduction: Concept of Mathematical Sculpture

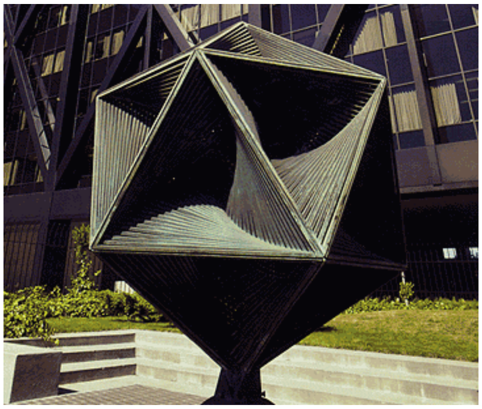
The main objective of this paper is to classify what I have called "mathematical sculpture". Therefore, I have to determine which kind of art I am trying to typify. I proposed the following definition: all the sculptures for which the use of Mathematics becomes essential in their conception, design, development or execution will belong to this typology, understanding Mathematics in its wider sense, from the simplest geometry to the most sophisticated calculus theorems. This definition is a little imprecise, because the word "essential" is a subjective concept. However, in most cases there are not any doubts about a specific work belonging to it, see figures 1 & 2. Bathsheba Grossman's "Metatrine" shows several properties related with Geometry (Polyhedrons) and Topology (Interwoven Figures). John Robinson, in his work "Dependent Beings", plays with the idea of Surface Orientation. All these concepts define different taxonomy groups or subgroups (I emphasise it by using initial capital letters).

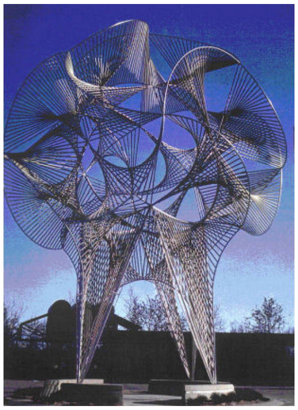

Figures 1 & 2 (from left to right). *"Metatrine" by Bathsheba Grossman, bronze, artist's web page. "Dependent Beings" by John Robinson, series "Universe", stone, artist's web page.*

Very different types of sculptures are included on mathematical sculpture, from works related from the simplest geometry to the most complex non-Euclidean geometry or topology. As regards their shapes, aspects, sizes and complexity a great diversity is possible. This is illustrated by the following examples: in figure 3 we can see a mathematically simple work in which Max Bill uses the simplest non-oriented surface, the Moebius strip; in figure 4 I show a complex work by Brent Collins which was designed with Sculpture Generator, a Carlo Sequin's software. It displays different configurations of "saddle rings" based on Scherk's Minimal Surfaces and curved to form the shape of a torus (ring); in figures 5 and 6 we show two sculptures by Charles Perry (we refer the reader to [2], where the sculptor analyses mathematical aspects of his work). Some sculpture is, of course, quite explicitly mathematical and represents surfaces and solids (figure 3). However, there is also sculpture that is implicitly mathematical and which does not strictly show mathematical elements, but uses mathematical ideas (for example, some abstract works by Eduardo Chillida, Richard Serra, Arthur Silverman). Consequently, this type of sculpture must be considered for classification as well. In view of the circumstances, it is difficult to draw a line between what is going to be accepted as mathematical sculpture or not.

I have divided mathematical sculpture into five main groups: Geometrical Sculpture, Sculpture with Concepts of Calculus, Sculpture with Concepts of Algebra, Topological Sculpture and Sculpture with Different Mathematical Concepts. I will try to avoid the conventional mathematical language, based on a complex algebraic formulation. Thus, such kind of language will be replaced by images of the sculptures.

Figures 3 & 4 (from left to right). *"Endless Ribbon" by Max Bill, 1953-56, Middleheim Outdoor Museum, Ambers, Belgium. "Hepteroid" by Brent Collins, Wood, 1997, photography by Philip Geller.*





Figures 5, 6 &7 (from left to right). *"Icosaspirale", San Francisco, USA and "Equinox", stainless steel, 1982, Lincoln Center, Dallas, USA; both works by Charles Perry, artist's web page. "Siglo XXI (XXIth Century)" by Pepe Noja, UPV Campus Site, Valencia, Spain.*

2. Research Development and educational purpose

First of all, I would like to say that the idea of researching into mathematical sculpture emerges in the last months of 2002, when I was browsing a book [3] that shows us the large sculpture collection of the UPV. Some items in this collection (figure 7 by Pepe Noja) can be considered mathematical. There are several studies on relationships between Mathematics and Sculpture that deal with specific aspects, such as the analysis of a particular sculptor or sculptures. However, there is no work in the scientific literature that provides a systematic analysis on this topic. Neither is there any study that offers an exhaustive classification. The scarcity and lack of studies led me to choose taxonomy as the main objective of my doctoral thesis supervised by Javier Barrallo. In May 2005 I got my PhD within UPV's Multidisciplinary Mathematics doctoral programme, with a thesis focused on mathematical sculpture witch was published as a complete book [4]. Finally, in 2006 we organised an exhibition of this type of sculpture sponsored by UPV's Vice-rectorate of Culture. At this event we exhibited the works of five mathematical sculptors: Bathsheba Grossman, Carlo Sequin, George Hart, Helaman Ferguson and Rinus Roelofs [5].

The educational purpose of classifying mathematical sculptures is to provide a more systematic approach in this field, so as to facilitate its incorporation in higher education. Courses devoted to the Mathematics-Art relationship are already being included in the academic content of artistic and technical studies syllabi. For instance, the School of Architecture (UPV) offers an optional subject on this topic.

3. Historical Backgrounds

I started the research for my doctoral thesis by checking relationships between Mathematics and Art that can be found from the dawn of mankind, in every kind of art and in most artistic manifestations, for instance, Mayan temples (figure 8). I went on analysing sculpture evolution in the last century, especially in Cubism, Constructivism, Abstract and Geometrical Art, Conceptualism and Minimalism. These movements played a decisive role in mathematical sculpture. As examples, I show a work (figure 9) made in 1913 by Umberto Boccione in which we observe the use of Geometry, a work (figure 10) made in 1937 by Mario Ceroli that could be considered a mathematical sculpture, and a whole conceptual mathematical work (figure 13) melted in 1951 by Max Bill. This sculpture appeared throughout the last century, mainly in its last decades. I also studied other general aspects: Conception Process, Manufactures Techniques Commanded by Computer and Current Situation and Prospects, etc.. This study could be faced from a common mathematical point of view, as I did with the work of some specific sculptors or pieces [4].

Figures 8, 9 &10. *Temple "Los Jaguares (Jaguars)", Mayan ruins of Chichén Itsá, Yucatan, Mexico, wash drawing by Adela Breton. "Unique Forms of Space Continuity" by Umberto Boccione, 1913, bronze. "In Balance" by Mario Ceroli, 1938, wood, Wadswhorh Atheneum, Conneticut, USA.*

4. Classifications of Mathematical Sculpture

The only approach I know to classify is based on the construction materials used, because they confer the works some geometrical properties. These types are wood, welded metal, concrete and stone. However, the kind of typology that I propose is based on mathematical properties. In our first approach to classification [1] the groups included were defined by a geometrical or mathematical property or concept, or by a combination of both, that characterised them. The types considered were: Classic and Polyhedral Geometry, Non-oriented Surfaces, Topological Knots, Quadric and Ruled Surfaces, Modular and Symmetric Structures, Boolean Operations, Minimal Surfaces, Transformations and Others. This first approach has been improved in my final thesis proposal which is based on different areas of Mathematics:

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- **Geometrical Sculpture**
  - Polyhedral
  - Curved Mathematical Surfaces
    - Quadrics
    - Revolution Surfaces
    - Ruled Surfaces
    - Other Surfaces
  - Non-Euclidean Geometries
  - Fractal Geometry
- **Sculpture with Concepts of Calculus**
  - Sequences and Mathematical Series
  - Sculptures with Concepts of Differential Calculus
    - Minimal or Zero-Mean Curved Surfaces
    - Other Concepts of Differential Calculus
  - Sculptures with Diverse Concepts of Calculus
- **Sculpture with Algebraic Concepts**
  - Symmetries
  - Transformations
  - Modular Sculptures
  - Boolean Operations
- **Topological Sculpture**
  - Non-Orientable Surfaces
  - Knots and Interwoven Figures
  - Other Topological Concepts
- **Sculpture with Different Mathematical Concepts**

The limits between these groups are not very strict, which is not surprising, since the division of Mathematics into different parts is sometimes not so clearly defined. I think that it is not possible to make a strict typology for mathematical sculpture. I have tried to classify each work according to its "dominant characteristic". This seems to happen in most works, which then can be included in one of the subgroups.



Figures 11, 12 &13 (from left to right). *"Unit Tripartite" by Max Bill, Award of Sculpture "1° Bienal de Sao Paulo", 1951, Brazil. "Fire & Ice" by George Hart, Oakwood and bronze, 1997, Vorpal Gallery, artist's web page. "Balletic Suite" by Brent Collins, photography by Philip Geller.*

5. Study of the Group and Subgroups Included in Mathematical Sculpture Classification

Some sculptures explicitly show their mathematical nature; a clear example could be a work based on the figure of a polyhedron or other specific geometrical shape (figure 12), so it will be easier classifying it. However, in other works mathematics is only present in an implicit or hidden way, such as figure 13, in which the mathematical conception is implicit in the design. I go on with my philosophy and introduce a description of the different groups and subgroups, giving examples to illustrate their main characteristics.

Geometrical Sculpture. This is the widest main group, as a consequence of the relationship between plastic arts, particularly Sculpture, and Geometry. There are many works that can be included (figures 5, 6 and 10). It is a type of sculpture with a great tradition, especially in the 20th century. By the beginning of this century we find some works in Cubism. Also, some authors belonging to Abstract, Minimal and Conceptual movements used Geometry as well. The following subgroups are included in this group:

Polyhedral Sculpture. The Platonic Polyhedrons are one of the solids most widely used by sculptors due to their beauty and simplicity. The truncated polyhedrons and a specific case, the Archimedean or semi-regular polyhedrons, are commonly used as well. The transformations on these solids, such as deforming, star-shaping or rounding their sides, or any other that may result in aesthetic effects, are interesting. As an example, see figure 15, based on a dodecahedron but with its sides replaced by 5-point stars. The work of George Hart [6] is a characteristic example of the huge number of variations that polyhedrons allow. A clear case is figure 14 which shows a sculpture that has the symmetry group of an orientated icosahedron.

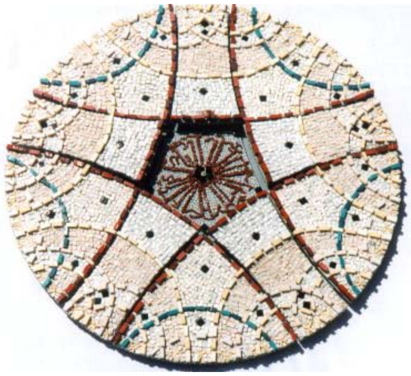
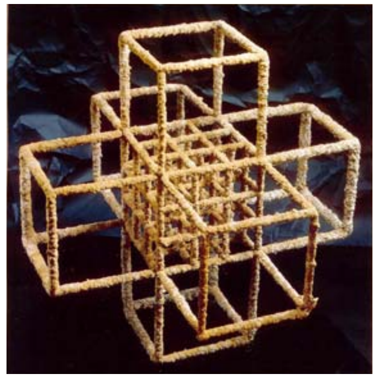
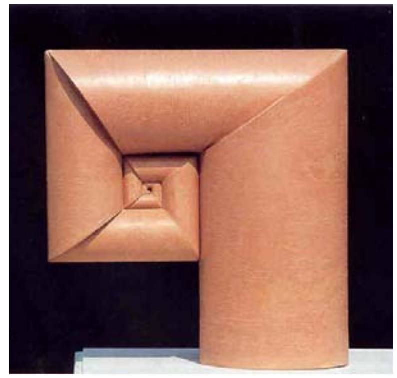
Mathematical Curved Surfaces. I have sub-divided it into non-excluding types: Quadrics, Revolution Surfaces, Ruled Surfaces and Other Surfaces. A commonly used surface is the hyperbolic paraboloid, which is a Quadric and a Ruled Surface simultaneously. I classify figure 6 as Ruled Surface. However, figure 16 is a Revolution Surface. Types are almost unlimited; I suggest surfaces of complex functions.

Non-Euclidean Geometries. I firmly believe that the use of these geometries (elliptic and hyperbolic) can be widely developed in Sculpture, as it happens in Painting. The most characteristic case is Escher. Irene Rosseau's sculpture (figure 17) is a good example of hyperbolic geometry that uses Poincare disc.

Fractal Geometry. Nowadays, the use in Mathematical sculpture of "new geometries" like fractal, different to the classical Euclidean, is not widespread. I show, as an example of the few sculptural works belonging to this subgroup I have found, a work by the Mexican sculptor Sebastian, in figure 18.



Figures 14, 15 & 16 (from left to right). *"Through" by Philip King, 1965, artist's collection, Bedfordshire, United Kingdom. "Stars Burst" by John Robinson, stainless steel, 1996, artist's web page. "Pareja (Couple)" by Carmen Grau, bronze, 2000, UPV Campus Site, Valencia, Spain.*

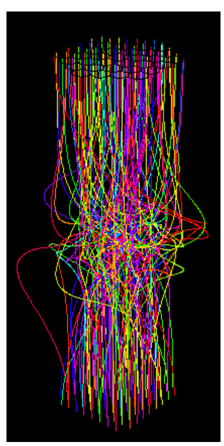
Figures 17, 18 & 19 (from left to right). *"Hyperbolic Diminution Sculpture I" by Irene Rousseau, wood cover with pieces of marble and glass, artist's web page. "Fractal Cube" by Sebastian, metal with aging treatment by saline cover, 1993-95, Mexico D.F. "Espiral Dos Raiz de Dos (Spyral Two Square Root of Two)" by Javier Carvajal, Exhibition of Bancaixa Foundation, 1991-94, Valencia, Spain.*

Sculpture with Concepts of Calculus. I will sort into this main group those works in which some concepts, processes and/or methods of Calculus are heavily implicated. I think that the possibilities of Calculus to develop sculpture have not been fully exploited. The subgroups included in this group are:

Sequences and Mathematical Series. I will include works in which their design implies the Calculus of Sequences and Series. For instance, this has been used for choosing measures, positions, etc., of the pieces that make up sculptures, such as in figure 19, a work by the Spanish sculptor Javier Carvajal.

Sculpture with Concepts of Differential Calculus. It is divided in Other Concepts of Differential Calculus and Minimal Surfaces or Zero-Mean Curvature; that is, local-area minimising surfaces resulting from the adoption of the minimum possible value of area for the given boundary curve (figure 4 shows a Scherk's minimal surface). Helaman Ferguson has also used them (figure 20 is based on Costa's surface).

Sculpture with Diverse Concepts of Calculus. This heterogeneous subgroup includes works deeply influenced by calculus concepts not belonging to previous subgroups. To illustrate the future possibilities that some Calculus methods can offer to sculpture design, I suggest interpolation (figure 21).

Figures 20, 21, 22 & 23 (from left to right). *"Costa II" by Helaman Ferguson, bronze, artist's web page. "Example of random interpolation image of Mathematica software help. "Custodia (Monstrance)" by Sebastian, silver, 2003. "Pascua (Easter)" by Michael Warren, UPV Campus Site, Valencia, Spain.*

Sculpture with Algebraic Concepts. This main group comprises works that make use of some algebraic concepts, processes and/or methods. Most sculptures can also adopt some geometric figures included in other groups, but if the algebraic property is the dominant aspect, then I will classify them within this group. I have divided it into Symmetries, Transformations, Modular Sculptures and Boolean Operations.

Symmetries. One of the algebraic properties with more applications in Art is symmetry. It is especially notorious in Architecture. In mathematical sculpture its utilisation is also rather usual. That is the reason why I add this subgroup [7]. Figure 21 is an example of a work with different symmetries.

Transformations. There are sculptures made with a mathematical solid (or a set of them) in which some algebraic transformation, such as movements, rotations and/or translations, have been applied. Because of the paper size limitation no example is included. I show works of this subgroup in an electronic paper [7].

Modular Sculptures. In this subgroup I classify works in which a given pattern, a motive of "mathematic type", is successively repeated. For instance, the very simple work by Michael Warren in figure 23.

Boolean Operations; that is, operations that fulfill the properties of Boolean Algebra. The works created using diverse transformations of the shape of one or several solids, based on Boolean Algebra, will be included in this subgroup. An example can be seen in figure 24, a sculpture by Bruce Beasley.

Topological Sculpture. This is the last main group of our classification that is base on a specific area of Mathematics: Topology. This subject deals with properties that are not affected by continue deformations, such as "flexion", "stretching" and "warping". Most important mathematical sculptors have made works of this type with very different designs. The subgroups included in Topological Sculpture are:

Non-Oriented Surfaces. These shapes are characterised by a vector calculus concept. However, because these surfaces have interesting topological properties, I sort into this subgroup. Max Bill, a pioneer in mathematical sculpture, extensively used the simplest non-oriented surface, the Moebius strip (figure 3). I have already included other works of this subgroup (figures 2 & 11). Figure 25 displays a last example.

Knots and Interwoven Figures. Mathematicians have studied "knots" for many centuries. This category of fascinating topological objects presents a wide range of possibilities to be used in Sculpture. Most mathematical sculptors have made use of them. Also, some of the best sculptors execute their works with Interwoven Figures (figure 26 by Keizo Ushio). Other examples are figures 4 & 7.

Figures 24, 25, 26 & 27 (from left to right). *"Solid Sequence" by Bruce Beasley, bronze, 1993, artist's web page. "Oushi Zokey, 2000", Bondi Exhibition, Sidney, Australia and "Oushi Zokey, 1999", San Sebastian, Spain; both works by Keizo Ushio, Carlo Sequin's web. Virtual Sculpture by Javier Barrallo.*

Other Topological Concepts. Sculptures of this subgroup are very heterogeneous, although with a deep influence of Topology (figure 11). Some pieces executed by Carlo Sequin (figure 28) are good examples.

Sculpture with Different Mathematical Concepts. Although I improved my taxonomy and added more groups, it would be impossible to cover all types of mathematical sculptures. It may also happen that a work falls into different groups because it does not show a unique "dominant mathematical concept". An example could be some Helaman Ferguson's works, such as the famous "Umbilic Torus NC" (figure 29). Also, I classify sculptures which are not so "explicitly mathematical", but in which maths is involved in their conception, for instance, the Nathaniel Friedman's work (figure 30). The computers have made possible to create mathematically complex works, such as Bathsheba Grossman's sculpture (figure 31).

Figures 28, 29, 30 & 31. *"Totem 3" by Carlo Sequin, bronze, 2003, artist's web page. "Umbilic Torus NC" by Helaman Ferguson, silicon bronze, artist's web page. "Trefoil Torso" by Nathaniel Friedman, limestone, artist's collection, shown in his web page. "The Unit Cube" by Bathsheba Grossman, silver and pedestal of black marble, artist's web page.*

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References

[1] Zalaya, Ricardo & Barrallo, Javier. *Mathematical Sculpture Classification*. Proceedings *Meeting Alhambra. ISAMA-BRIDGES 2003*, pp. 53-60. I.S.B.N.: 84-930669-1-5. Granada, 2003.

[2] Perry, Charles O. *Mathematical Sculpture*. Proceedings *Mathematics & Design 98, II Congress International*, pp. 367-384. I.S.B.N.: 84-600-9459-6. San Sebastián, 1997.

[3] Vicerectorat de Cultura Universidat Politècnica de València. *Esculturas del Campus de la Universidad Politécnica de Valencia*. I.S.B.N.: 84-9705-117-3. Valencia, 2001.

[4] Zalaya, Ricardo & Barrallo, Javier. *La Escultura Matemática y su Clasificación*. I.S.B.N.: 84-8363-044-3. Valencia, 2006.

[5] Vicerectorat de Cultura Universidat Politècnica de València. *Esculturas Matemáticas*. Catalogue of a exhibition of title *Mathematical Sculpture*. I.S.B.N.: 84-8363-043-5. Valencia, 2006.

[6] Hart, George W. *Sculpture from Symmetrically Arranged Planar Components*. Proceedings *Meeting Alhambra. ISAMA-BRIDGES 2003*, pp. 315-322. I.S.B.N.: 84-930669-1-5. Granada, 2003.

[7] Zalaya, Ricardo & Barrallo, Javier. *Classification of Mathematical Sculpture*. *Visual Mathematics* (electronics magazine). Vol. 6, Number 4, 2004.